Optimal advertising-pricing decisions for national and private label competition in a dynamic framework

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Abstract:
In this paper, competition between a private label and a national brand is investigated in a marketing channel. We considered a marketing channel formed by one manufacturer and one retailer. The retailer sells the manufacturer’s brand and also sells her own private label. The manufacturer is considered as the leader of marketing channel, so the retailer plays the follower role in a Stackelberg differential game. Considering pricing decisions, advertising effort and promotion expenditure in the model provides a complete analysis of market conditions. The obtained results show the optimal marketing strategies including pricing and advertising for each of players. Eventually, a numerical example is presented to illustrate the applicability of the proposed model.

Keywords: Differential games; Marketing; Pricing; Advertising; Feedback Stackelberg equilibrium.

1. Introduction

As a result of increase in national brand prices, the market share of private labels (also called store brands) has been increased in recent decades (see [1], [2]). Hence, the competition between national and store brands has become an important issue for both manufacturers and retailers. Many studies have investigated the emergence effect of private labels in marketing channels. Several papers focused their attention on price competition between these brands (see e.g. [3], [4]). Some of these papers also analyzed the effects of advertising strategies on the competition (see e.g. [5], [6]). However some other papers investigated interaction effects of advertising and pricing strategies on the competition simultaneously (see e.g. [7], [8]).

The aim of this paper is to characterize pricing and advertising feedback strategies and to analyze the strategic interaction between these variables in competition between national and store brands. What makes our research different from the relevant literature is incorporating promotional activities of the retailer beside advertising and pricing decisions. For this purpose, we consider a marketing channel formed by one manufacturer and one retailer. The retailer sells the manufacturer’s brand and also sells his own private label. We consider the manufacturer as the leader of the market and the retailer as the follower player in a Stackelberg differential game (see [6]–[13] for examples of Stackelberg differential games).

Karray and Zaccour [6] presented a cooperative model in a same marketing channel; they supposed that retailer may promote national brand, and also considered a coop program on promotion which was implemented by the manufacturer. They assumed that the coop promotion is a cost sharing mechanism where the manufacturer pays part of the national brand promotion cost paid by retailer. In addition they supposed advertising and promotion as control variables. Characterizing pricing strategies as well as advertising and promotion makes a contribution toward this paper stream.

Amrouche et al. [7] also studied the relationship between pricing and advertising in a channel like Karray and Zaccour [6] model. They considered goodwill for both brands and supposed that each player can advertise for his own brand. Authors also assumed that retailer don’t promote for the store brand. Karray and Martín-Herrán [8] presented a model similar to Amrouche et al. [7]. They managed to solve their model analytically unlike Amrouche et al. [7]. Contrary to Karray and Martín-Herrán [8] and Amrouche et al. [7], we don’t consider any goodwill stock for private label, because if the store brand was able to afford any national advertise program and provide goodwill for himself, he wouldn’t be recognized as a private label. actually we suppose that the retailer
don’t have national advertising capability and can only encourage consumer to buy his own private label by local promotion.

The rest of the paper is structured as follows. In Sect. 2, we develop a model for the channel under our realistic consideration. Feedback-Stackelberg equilibrium of the proposed model is characterized in Sect. 3. Sect. 4 is dedicated to interpreting the numerical findings and discussing about the results. In Sect. 5, we conclude and suggest directions for future research.

2. Model

We consider a marketing channel formed by one manufacturer (m) and one retailer (r). The manufacturer product is distributed just by the retailer. The retailer sells two different goods, the manufacturer national brand (nb) and also his own store brand (sb) or private labels. We assume that the retailer products the store brand or afford it from a manufacturer at a constant cost C_{nb} (See Karray and Zaccour [6], Amrouche et al. [7] and Karray and Martín-Herrán [8]). The manufacturer also produces the national brands at a constant cost C_{nb}. The retailer controls the retail price of national brand (M_{nb}(t)), the retail price of store brand (M_{sb}(t)) and local promotion effort for private label (P_{sb}(t)). Manufacturer, as leader of marketing channel controls the wholesale price of his national brand (W_{nb}(t)) and national advertising effort of his brand (A(t)).

Consumers are impressed by these control variables and cohort of them at any instant time \( t \in [0, \infty) \) formed the dynamic demand function for each of two brands. The dynamic demand function for each brand is assumed to be linear and at any time \( t \) given by:

\[
Q_{nb}(t) = B_{nb} + \theta G(t) - \beta P_{sb}(t) - \omega_1 M_{nb}(t) + \varepsilon_1 M_{sb}(t) \\
Q_{sb}(t) = B_{sb} - \gamma G(t) + \alpha P_{sb}(t) - \omega_2 M_{sb}(t) + \varepsilon_2 M_{nb}(t)
\]  

(1)  

(2)

Where \( Q_{nb}(t) \) is national brand demand at time \( t \), \( Q_{sb}(t) \) is store brand demand at time \( t \), \( B_{nb} \) and \( B_{sb} \) is independent demand of each brand. \( G(t) \) is national brands goodwill. \( \theta, \gamma, \beta, \alpha, \omega_1, \omega_2, \varepsilon_1 \) and \( \varepsilon_2 \) are positive parameters.

As mentioned in section 1, we don’t consider any goodwill for the store brand and suppose that the retailer don’t have national advertising capability and only can encourage consumer to buy his own private label by local promotion. The national brand stock of accumulated advertising goodwill dynamics is followed by Nerlove and Arrow [14]:

\[
\dot{G}(t) = AA(t) - \delta G(t), \quad G(0) = 0
\]

(3)

Where \( \lambda \) is a positive parameter and \( \delta > 0 \) is the decay rate (also called depreciation rate).

The advertising and promotion costs for players are defined by a convex quadratic function (see e.g. [12], [15]). Then the net national brand advertising cost for manufacturer and store brand promotion cost for retailer are as follow:

\[
C_m(t) = \frac{1}{2} U_m A^2(t)
\]

(4)

\[
C_r(t) = \frac{1}{2} U_r P_{sb}^2(t)
\]

(5)

Where \( U_m \) and \( U_r \) are positive parameters, \( C_m(t) \) is advertising effort cost for manufacturer and \( C_r(t) \) is promotion effort cost for retailer. We also assume that \( U_m > U_r \) because national advertising costs is more expensive than local promotion cost.

We denote discount rate by \( \rho \). There is no cooperation program and each of players wants to maximize his own discounted profit over infinite time horizon, \( \pi_i \in \{m, r\} \). So the optimization problems of manufacture (m) as leader and the retailer (r) as follower in the Stackelberg differential games are as follows:

\[
\max_{\pi_m} \pi_m = \int_0^\infty e^{-\rho t} \left[ (W_{nb}(t) - C_{nb})(Q_{nb}(t)) - C_m(t) \right] dt
\]

\[
= \int_0^\infty e^{-\rho t} \left[ (W_{nb}(t) - C_{nb})(P_{nb} + \theta G(t) - \beta P_{sb}(t) - \omega_1 M_{nb}(t) \\
+ \varepsilon_1 M_{sb}(t)) - \frac{1}{2} U_m A(t)^2 \right] dt
\]

(6)
\[
\max_{\pi_r} \pi_r \quad = \int_0^\infty e^{-\rho t} \left[ (M_{nb}(t) - W_{nb}(t))(Q_{nb}(t)) + (M_{sb}(t) - C_{sb})(Q_{sb}(t)) - C_r(t) \right] dt
\]

\[
M_{nb}, M_{sb}, P_{sb} = \int_0^\infty e^{-\rho t} \left[ (M_{nb}(t) - W_{nb}(t))(Q_{nb}(t)) + (M_{sb}(t) - C_{sb})(Q_{sb}(t)) - C_r(t) \right] dt
\]

Subject to:
\[G(t) = \lambda A(t) - \delta G(t), \ G(0) = 0\]

We consider a usual assumption in the case of infinite time horizon differential games; the game is played under a feedback information structure. The solution of this problem and optimal player’s control variables is obtained in section 3.

### 3. Feedback Stackelberg equilibrium

In order to solve the models, a formal Stackelberg model solution procedure is followed. First, the leader estimates the retailer’s reaction functions and injects this information into his objective function and then, determines his optimal control variables and feedback strategies. Finally by replacing the leader equilibrium control variables in the followers’ reaction functions, the optimal strategies of follower are characterized. The feedback Stackelberg equilibrium for the model is obtained in the following:

In order to obtain the feedback equilibrium strategies for the model, we follow the optimality principles of dynamic programming. According to the solution, the manufacturer (Leader) optimal pricing and advertising is characterized as:

\[
A(t) = \frac{\lambda}{U_m} (T_1 G(t) + T_2)
\]

\[
W_{nb}(t) = \frac{s_{12} G(t) + s_{13}}{s_{11}}
\]

Retailer (Follower) optimal pricing and promotion strategies are characterized as:

\[
M_{nb}(t) = \frac{s_2 G(t) + s_3 s_{13} + s_{12} G(t) + s_4}{s_{11}}
\]

\[
M_{sb}(t) = \frac{s_6 G(t) + s_6 s_{13} + s_{12} G(t) + s_7}{s_{11}}
\]

\[
P_{sb}(t) = -\frac{s_8 G(t) + s_9 s_{13} + s_{12} G(t) + s_{10}}{s_{11}}
\]

Where G(t) = \frac{\lambda^2}{\lambda^2 + \rho^2}(1 - e^{-\frac{T_1 + \rho T_2}{U_m}}). For clarity of presentation, we include the expressions of the coefficients S_i = 1, 2, ..., 15 and T_1, T_2 in Appendix 1.

Proof. In order to solve the model, first the retailer reaction functions should be computed. Since in our model the retailer control variables do not affect into the dynamicity the problem, we can solve the retailer strategies stastically. The problem that the retailer is facing is:

\[
\max_{M_{nb}, M_{sb}, P_{sb}} \left[ (M_{nb}(t) - W_{nb}(t))(Q_{nb}(t)) + (M_{sb}(t) - C_{sb})(Q_{sb}(t)) - C_r(t) \right]
\]

So from the necessary conditions for optimality, taking the partial derivative of the retailer objective function with respect to M_{nb}(t), M_{sb}(t) and P_{sb}(t) and equating each of them to zero, we face to a simple system of linear equations system including three equations and three variables:
\[
\begin{align*}
\theta G(t) - \beta P_{sb}(t) - \omega_1 M_{nb}(t) + \epsilon_1 M_{sb}(t) + B_{nb} - \omega_1 (M_{nb}(t) - W_{nb}(t)) + \varepsilon_2 (M_{sb}(t) - L) \\
- \gamma G(t) + \alpha P_{sb}(t) + \varepsilon_2 M_{nb}(t) - \omega_2 M_{sb}(t) + B_{sb} + \epsilon_1 (M_{nb}(t) - W_{nb}(t)) - \omega_2 (M_{sb}(t) - L)
\end{align*}
\]

By solving above linear system of equations, the retailer reaction functions are computed as:
\[
\begin{align*}
M_{nb}(t) &= \frac{1}{s_1} [s_2 G(t) + s_3 W_{nb} + s_4] \\
M_{sb}(t) &= \frac{1}{s_1} [s_5 G(t) + s_6 W_{nb} + s_7] \\
P_{sb}(t) &= \frac{1}{s_1} [s_8 G(t) + s_9 W_{nb} + s_{10}]
\end{align*}
\]

Now we are able to compute the manufacturer optimal strategies by replacing the retailer reaction function into the manufacturer problem and form the Hamilton–Jacobi–Bellman (HJB) equations for him. After substituting 19, and 20 and 21 into 6, the manufacturer objective function is characterized as:
\[
\begin{align*}
\max_{\pi_m, A(t), W_{nb}(t)} & \quad \int_0^\infty \left[ (W_{nb}(t) - C_{nb}) \left( B_{nb} + \theta G(t) - \beta \left( \frac{1}{s_1} (s_9 G(t) + s_9 W_{nb} + s_{10}) \right) \\
- \omega_1 \left( \frac{1}{s_1} [s_2 G(t) + s_2 W_{nb} + s_4] \right) + \varepsilon_1 \left( \frac{1}{s_1} [s_5 G(t) + s_6 W_{nb} + s_7] \right) \right) \\
- \frac{1}{2} U_m A(t)^2 \right] dt
\end{align*}
\]

Given the manufacturer’s objective function, the Hamilton–Jacobi–Bellman (HJB) equation can be specified as:
\[
\begin{align*}
\rho V_m &= \max \left[ (W_{nb}(t) - C_{nb}) \left( B_{nb} + \theta G(t) - \beta \left( \frac{1}{s_1} (s_9 G(t) + s_9 W_{nb} + s_{10}) \right) \\
- \omega_1 \left( \frac{1}{s_1} [s_2 G(t) + s_2 W_{nb} + s_4] \right) + \varepsilon_1 \left( \frac{1}{s_1} [s_5 G(t) + s_6 W_{nb} + s_7] \right) \right) \\
- \frac{1}{2} U_m A(t)^2 ] + \frac{\partial}{\partial G} V_m \left( \lambda A(t) - \delta G(t) \right) \right]
\end{align*}
\]

Solving the manufacturer HJB equation, the manufacturer’s optimal pricing and advertising strategies are obtained as equations 12 and 13. We get two possible expressions for the equilibrium advertising strategies after solving the HJB equations for the manufacturer. Because advertising strategies has to converge in the problem time horizon, only one of the two solutions is eligible and we choose the one that leads to stable steady (see Appendices 1 for detailed computations of the results).

The retailer’s optimal pricing and promotion strategies is characterized as equations 14, 15 and 16 by replacing the manufacturer equilibrium variables into the retailer reaction functions.

4. Comparison

Although we analytically calculate equilibrium for the model, but unfortunately we cannot reach into an explicit analytical comparison between the two equilibrium, since the obtained feedback strategies functions are heavily complex. So, in order to make a clear comparison, we consider a practical numerical example and determine values of the obtained equilibrium in both of the models. We assume the following values for the parameters as a benchmark case:

Demand Parameters: \( B_{nb} = 1000, B_{nb} = 500, \theta = 3, \gamma = 1.3, \alpha = 3, \beta = 1.5, \omega_1 = 8, \omega_2 = 10, \varepsilon_1 = 3, \varepsilon_2 = 3 \)

Cost Parameters: \( U_m = 1.6, U_r = 1.2, C_{nb} = 20, C_{sb} = 25 \)

Dynamic Parameters: \( \lambda = 0.7, \delta = 0.45 \)

Discount Rate: \( \rho = 0.03 \)
Plugging these values into the equilibria of the model yields results described below. Note that in order to make a clear sense about dynamic behavior of feedback strategies; we plot the dynamic variation of these variables over time. Hence we skipped presenting numerical values of control variables in the defined equilibrium functions.

4.1. Pricing:

Figure 1 shows the optimal strategies for three price variables in the model; including wholesale price of national brand $W_{nb}(t)$ and retail prices of store and national brands, $M_{sb}(t), M_{nb}(t)$.

![Figure 1: Optimal pricing strategies for each player](image)

4.2. Promotion and Advertising:

In order to show the optimal advertising strategies, we plot the optimal values of advertising and promotion strategies for both Players in Figure 2.

![Figure 2: Optimal advertising and promotion effort of both players](image)

As shown in figure 2 the manufacturer as the leader of market must put more effort into advertising compared to the retailer.

4.3. Demand:
Figure 3 shows demand variation of both brands over time. Comparing results indicates that the national brand demand increases significantly during the time horizon.

4.4. Discounted benefits:

Discounted benefit of the manufacturer and the retailer is shown in Figure 4. As shown in Figure 4, the manufacturer gains a significant benefit compared to the retailer if he follows the optimal strategies.

5. Concluding Remarks
In this paper, competition between a private label and a national brand was investigated in a marketing channel. For this purpose, we considered a marketing channel formed by one manufacturer and one retailer. The retailer sells the manufacturer brand and also sells a private label. The manufacturer controls the dynamic wholesale price and its own advertising rate, the retailer also has three control variables, including retail prices of both national brand and private label and promotion rate of national brand. It’s worth noting that we considered positive and negative effects and impacts of these variables on the manufacturer and the retailer benefits.

We assumed the players in a Stackelberg differential game where the manufacturer is the leader of the market and the retailer play follower role in the game. As the result, the optimal marketing strategies including pricing and advertising are developed for each of channel members. Eventually, results of the model are compared using a numerical example.

Other extensions to this work can be expected. First, one could consider goodwill level for store brand and allow the retailer to advertise his own brand. Another option is to assume another national brand in the marketing channel as an exclusive representative of the national brand and analyze the effects of existence of a new manufacturer on the marketing channel. Finally, one could also consider a new retailer in the marketing channel as an exclusive representative of the national brand and analyze the competition in the marketing channel in this mode.

Appendix 1. Computation of the feedback equilibria

As mentioned in section 3.1, equations 19, 20 and 21 shows the retailer’s reaction functions. For clarity of presentation we define $s_i$ $i = 1, ..., 10$ in these equations as follow:

$$S_1 = -[2a^2e_1 + 2B^2e_2 - 2a\beta(e_1 + e_2) + U_r((e_1 + e_2)^2 - 4\alpha e_1e_2)]$$

$$S_2 = -\gamma U_r(e_1 + e_2) + 20U_r^2e_2 + a^2\theta + a\beta \gamma$$

$$S_3 = -U_r(e_1^2 + e_1e_2 - 2\alpha e_1e_2) + a\beta(2e_1 + e_2) - a^2e_1 - 2B^2e_2$$

$$S_4 = C_{sb}(U_r e_0^2 - U_r e_1^2 - 2e_1 + 2\alpha e_1e_2 + B_{sb}(U_r e_1 + U_r e_2 - a\beta) + B_{nb}(-a^2 + 2U_r e_2)$$

$$S_5 = \theta(U_r e_1 + U_r e_2 - a\theta) + \gamma(\beta^2 - 2U_r e_1)$$

$$S_6 = \omega_1(a\beta - U_r e_1 + U_r e_2) - \beta^2e_2$$

$$S_7 = C_{sb}(-2a^2e_1 + a\beta e_1 + 2\alpha \beta e_2 - e_1 + e_1^2 - U_r e_1 e_2 - U_r e_2^2 + 2U_r e_0 e_0 + B_{nb}(U_r e_1 + U_r e_2 - a\beta) + B_{sb}(U_r e_1 + U_r e_2 - a\beta)$$

$$S_8 = \alpha(2\gamma e_1 - \theta e_1 - \theta e_2) + \beta(2\theta e_2 - \gamma e_1 - \gamma e_2)$$

$$S_9 = \alpha(2\gamma e_1 - \theta e_1 - \theta e_2) + \beta(2\theta e_2 - \gamma e_1 - \gamma e_2)$$

$$S_{10} = C_{sb}(2\alpha \omega_1 e_2 - a\beta e_2 - a\beta e_2 - \beta e_1 e_2 + B_{nb}(2\beta e_1 - \alpha e_1 - \alpha e_2) + B_{sb}(\beta e_1 + \beta e_2 - 2\alpha e_1)$$

Anticipating the retailer’s response in 19, 20 and 21, the manufacturer’s HJB equation is given by:

$$\rho V_m = \max \left[ \left( \left( W_{nb}(t) - C_{nb} \right) (B_{nb} + \theta G(t)) - \beta \left( \frac{1}{S_1} (s_2 G(t) + s_9 W_{nb} + s_10) \right) \right) \right.$$  

$$\left. \left. - \omega_1 \left( \frac{1}{S_1} [s_2 G(t) + s_3 W_{nb} + s_4] \right) \right] + e_1 \left( \frac{1}{S_1} [s_5 G(t) + s_6 W_{nb} + s_7] \right) \right)$$

$$\left. - \frac{1}{2} U_m A(t)^2 + \frac{\partial}{\partial G} V_m (\lambda A(t) - \delta G(t)) \right]$$

Which yields below the optimal feedback decision for the manufacturer:

$$W_{nb} (t) = \frac{1}{S_{11}} (s_{12} G(t) + s_{13})$$

$$A(t) = \frac{\lambda}{U_m} \frac{\partial V_m}{\partial G}$$

Where:

$$S_{11} = -2 (\beta s_9 + \varepsilon_1 s_6 - \omega_1 s_2)$$

$$S_{12} = \beta s_9 + \theta s_1 + \varepsilon_1 s_5 - \omega_2 s_2$$

$$S_{13} = -\beta C_{nb} s_9 - C_{nb} \varepsilon_1 s_6 + C_{nb} \omega_1 s_3 + \beta s_{10} + B_{nb} s_1 + \varepsilon_1 s_7 - \omega_1 s_4$$

Substituting (A2) and (A3) into (19), (20) and (21) produces:

$$M_{nb} (t) = \frac{s_{22} G(t) + s_3 + s_{12} G(t)}{s_{11}} + s_4$$

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\[ M_{sb}(t) = \frac{s_5 G(t) + s_6 \frac{s_{13} + s_{12} G(t)}{s_{11}} + s_7}{s_1} \quad (A5) \]

\[ P_{sb}(t) = -\frac{s_8 G(t) + s_9 \frac{s_{13} + s_{12} G(t)}{s_{11}} + s_{10}}{s_1} \quad (A6) \]

Subject to (A2), (A3), (A4), (A5) and (A6), the feedback equilibrium corresponds to the solution of the partial differential equation in (A1). Conjecture the following quadratic functions as the solution to (A1):

\[ V_m = \frac{1}{2} T_1 G^2(t) + T_2 G(t) + T_3 \quad (A7) \]

It follows from (A7) that:

\[ \frac{\partial V_m}{\partial G} = T_1 G(t) + T_2 \quad (A8) \]

After substituting (A8) into (A3) and then into (3), the state equation can be specified as:

\[ \dot{G}(t) = \frac{\lambda^2}{U_m} (T_1 G(t) + T_2) - \delta G(t) \quad (A9) \]

Solving the differential equation in (A9) with the initial condition \( G(0) = 0 \) results in:

\[ G(t) = \frac{\lambda^2 T_2}{-\lambda^2 T_1 + \delta U_m} (1 - e^{-\frac{\lambda^2 T_1 + \delta U_m t}{U_m}}) \quad (A10) \]

Plugging (A2), (A3), (A4), (A5), (A6), (A7) and (A8) into (A1) and then equating the coefficients of \( G^2 \) and \( G \) on both sides of that gives:

\[ T_1 = \frac{1}{\rho U_m} (\lambda^2 T_1^2 - 2\delta U_m T_1 + S_{14}) \quad (A11) \]

\[ T_2 = \frac{1}{\rho U_m} (\lambda^2 T_1 T_2 - \delta U_m T_2 + \frac{S_{15}}{2s_{11}^2}) \quad (A12) \]

where

\[ S_{14} = \frac{U_m}{s_{12}^2} (2\beta s_3 s_{12} + 2\epsilon_1 s_6 s_{12}^2 + 2\omega_1 s_{13} s_{12}^2 + 2\beta s_8 s_{11} s_{12} + 2\theta s_1 s_{11} s_{12} + 2\epsilon_1 s_5 s_{11} s_{12} - 2\omega_1 s_2 s_{11} s_{12} \]

\[ S_{15} = 2U_m (-\beta C_{nb} s_9 s_{11} s_{12} - C_{nb} \epsilon_1 s_6 s_{11} s_{12} + C_{nb} \omega_1 s_3 s_{11} s_{12} - \beta C_{nb} s_8 s_{11}^2 - \theta C_{nb} s_1 s_{11}^2 - C_{nb} \epsilon_1 s_5 s_{11}^2 + C_{nb} \omega_1 s_2 s_{11}^2 + \beta s_8 s_{11} s_{13} + 2\beta s_9 s_{12} s_{13} + \beta s_{10} s_{11} s_{12} + \theta s_1 s_{11} s_{13} + B_{nb} s_1 s_{11} s_{12} + \epsilon_1 s_5 s_{11} s_{12} + \epsilon_1 s_6 s_{12} s_{13} + \epsilon_1 s_7 s_{11} s_{12} - \omega_1 s_2 s_{11} s_{13} - 2\omega_1 s_3 s_{12} s_{13} + 2\omega_1 s_4 s_{11} s_{12}) \]

Which leads to two possible solutions of \( T_1 \). Because \( G(t) \) specified in (A10) has to converge in \( t \), the solution to (A11) must satisfy \( \lambda^2 T_1 - \delta U_m < 0 \). Only one of the two solutions is eligible. The unique one is:

\[ T_1 = \frac{2\delta U_m + \rho U_m - \sqrt{4\delta^2 U_m^2 + 4\delta \rho U_m^2 + \rho^2 U_m^2 - 4\lambda^2 S_{14}}}{2\lambda^2} \quad (A13) \]

Substituting (A13) into (A12) yields:

\[ T_2 = \frac{S_{15}}{s_{11}^2 (\rho U_m + \sqrt{4\delta^2 U_m^2 + 4\delta \rho U_m^2 + \rho^2 U_m^2 - 4\lambda^2 S_{14})}} \quad (A14) \]

Substituting (A13) and (A14) into (A10) and then into (A2) and (A3) yields the manufacturer’s optimal pricing and advertising strategies (12) and (13). The retailer’s optimal promotion and pricing strategies (14), (15) and (16) can be obtained by substituting (A13) and (A14) into (A10) and then into (A4), (A5) and (A6)

References